$$D_0(x) = \partial_0 + ieA_0(x) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]0}(x),$$

$$D_k(x) = e^j_{(k)}(x)(\partial_j + ieA_j(x)) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]k}(x), \; k = 1, 2, 3.$$

The definition of the 6 large components remains the same, two linear constraints preserve their form as well. All algebraic transformations proving existence of only 4 independent equations also are the same. The difference consists only in the new and more complicated expressions for generalized derivatives. Correspondingly, we obtain the generalized equation

$$iD_0(x)\Psi = -\frac{1}{2M}(D_1^2(x) + D_2^2(x) + D_3^2(x))\Psi + \frac{1}{2M}(D_{[23]}S_1 + D_{[31]}S_2 + D_{[12]}S_3)\Psi_4,$$

where the commutators $D_{[kl]} = D_k(x)D_l(x) - D_l(x)D_k(x)$ are used.

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SPIN 3/2 PARTICLE IN THE COULOMB FIELD, TETRAD FORMALISM, NONRELATIVISTIC APPROXIMATION

In the present paper, we will study the nonrelativistic approximation for a spin 3/2 particle [1–3] in the external Coulomb field. In [4] (see also [5; 6]), the nonrelativistic equation was derived from the relativistic Pauli – Fierz theory; in this case, the Cartesian coordinates were used and the presence of an arbitrary electromagnetic field was taken into account. In the present paper, the system of radial equations describing the nonrelativistic spin 3/2 particle in the Coulomb field will be obtained. Exact solutions are found in terms of the confluent hypergeometric functions, and the corresponding energy spectra are obtained.

In [4; 5], the system of radial equations for spin 3/2 particle in absence of external fields was derived. The substitution for the wave function has the form [6]

$$\Psi = \Psi_{A(l)} = e^{-i\epsilon t}\begin{vmatrix} f_0 D_{-1/2} & f_1 D_{-3/2} & f_2 D_{-1/2} & f_3 D_{+1/2} \\ g_0 D_{+1/2} & g_1 D_{-1/2} & g_2 D_{+1/2} & g_3 D_{+3/2} \\ h_0 D_{-1/2} & h_1 D_{-3/2} & h_2 D_{-1/2} & h_3 D_{+1/2} \\ d_0 D_{+1/2} & d_1 D_{-1/2} & d_2 D_{+1/2} & d_3 D_{+3/2} \end{vmatrix}, \quad (1)$$

where we use the Wigner D-functions, $D_\sigma = D^j_{-m,\sigma}(\phi,\theta,0);\ j = 1/2, 3/2, 5/2, \ldots$. From diagonalization of the space reflection operator, we get the following restrictions (where $\delta = \pm 1$):

$$d_0 = \delta f_0\,,\ d_1 = \delta f_3\,,\ d_2 = \delta f_2\,,\ d_3 = \delta f_1, \quad h_0 = \delta g_0\,,\ h_1 = \delta g_3,\ h_2 = \delta g_2\,,\ h_3 = \delta g_1;$$

these restrictions preserve only 8 independent functions, so the substitution (1) is simplified. After separating the variables, in [4; 5] where found 8 equations; they contain parameters $a = j + 1/2,\quad b = \sqrt{(j-1/2)(j+3/2)}$; the equations for states with opposite parities differ only in sign at the mass, so we follow only the case $\delta = +1$; we omit their explicit form.

There is the well-known method for obtaining the nonrelativistic approximation in equations for spin 3/2 particle in Cartesian [7]

$$[Y^0\partial_0 + iY^1\partial_1 + Y^2\partial_2 + Y^3\partial_3 + iM]\Psi^{cart} = 0.$$

On the base of 3-order minimal equation for the matrix Y^0, we introduce three projective operators that allow us to decompose the wave function into the sum of three components: one large and two small. In order to apply the same method in spherical coordinates, we need explicit form for three matrices (see notations in [4; 5])

$$Y^0 = \Gamma^{-1}\Gamma^0, \quad \Gamma^0 = \gamma^5\left(\gamma^1 \otimes \mu^{[01]} + \gamma^2 \otimes \mu^{[02]} + \gamma^3 \otimes \mu^{[03]}\right),$$

$$\Gamma = \gamma^5\{s_{01} \otimes \mu^{[01]} + s_{02} \otimes \mu^{[02]} + s_{03} \otimes \mu^{[03]} + s_{23} \otimes \mu^{[23]} + s_{31} \otimes \mu^{[31]} + s_{12} \otimes \mu^{[12]}\}.$$

Let us find the 16-dimensional representation for the matrix Y^0. Its minimal equation is $Y_0^2(Y_0^2 - 1) = 0$. We can introduce three projective operators

$$P_0 = 1 - Y_0^2, \quad P_+ = +Y_0^2(Y_0 + 1)/2, \quad P_- = -Y_0^2(Y_0 - 1)/2.$$

Below we need the explicit form of these operators, due to their bulkiness, we do not present it here. Further we obtain the following expressions for complete wave function (we introduce short notations for large and small components, and take into account the parity restrictions)

$$\Psi_{\delta=+1} = \begin{vmatrix} f_0 \\ g_0 \\ f_1 \\ g_1 \\ g_3 \\ f_3 \\ f_2 \\ g_2 \end{vmatrix} = \begin{vmatrix} S_1 \\ S_2 \\ L_1 + y_1 \\ L_2 + y_2 \\ L_1 - y_1 \\ L_2 - y_2 - \sqrt{2}y_4 - \sqrt{2}y_5 \\ \sqrt{2}L_2 + y_4 \\ \sqrt{2}L_2 + y_5 \end{vmatrix}.$$

The truncated 8-dimensional column is composed of functions included in the 8-dimensional radial system. We substitute these relations into the radial system derived in [4; 5];

in the same time we separate the rest energy by the formal change $m = (m+E),\ E << m;$ then after some linear recombining within equations we derive

$$\frac{2\sqrt{2}(a+5)L_2}{mr} + y_4\left(\frac{4(a-1)}{mr} + 2i\right) + y_5\left(\frac{4(a-1)}{mr} + 6i\right) + \frac{2\sqrt{2}bL_1}{mr} + \frac{4\sqrt{2}L_2'}{m} - \frac{4y_4'}{m} - \frac{4y_5'}{m} + 4i\sqrt{2}y_2 = 0,$$

$$\frac{2\sqrt{2}(a+3)y_2}{mr} + y_5\left(\frac{4(a+1)}{mr} + 6i\right) + \frac{2\sqrt{2}by_1}{mr} + \frac{4\sqrt{2}y_2'}{m} + \frac{4y_4'}{m} + \frac{4y_5'}{m} + \left(\frac{8}{mr} + 6i\right)y_4 = 0,$$

$$\frac{\sqrt{2}bS_1}{mr} + \frac{\sqrt{2}bS_2}{mr} + \frac{\sqrt{2}by_4}{mr} - \frac{\sqrt{2}by_5}{mr} - \frac{4iEL_1}{m} + \frac{4y_1'}{m} + \frac{4y_1}{mr} = 0,$$

$$\frac{4bL_2}{mr} + \frac{\sqrt{2}bS_1}{mr} - \frac{\sqrt{2}bS_2}{mr} + \frac{\sqrt{2}by_4}{mr} + \frac{\sqrt{2}by_5}{mr} + \frac{4L_1'}{m} + \frac{4L_1}{mr} - \frac{4i(2m+E)y_1}{m} = 0,$$

$$\frac{\sqrt{2}(a+1)S_1}{mr} + \frac{\sqrt{2}(a+1)S_2}{mr} - \frac{4ay_2}{mr} + \frac{\sqrt{2}(1-3a)y_4}{mr} - \frac{\sqrt{2}(a+1)y_5}{mr} - \frac{4by_1}{mr} - \frac{12iEL_2}{m} -$$
$$- \frac{2\sqrt{2}S_1'}{m} - \frac{2\sqrt{2}S_2'}{m} + \frac{4y_2'}{m} + \frac{2\sqrt{2}y_4'}{m} + \frac{2\sqrt{2}y_5'}{m} = 0,$$

$$\frac{4(a+2)L_2}{mr} + \frac{\sqrt{2}S_1(a-2imr-1)}{mr} - \frac{\sqrt{2}S_2(a+2imr-1)}{mr} + \frac{\sqrt{2}y_5(a+4ir(2m+E)-1)}{mr} + \frac{\sqrt{2}(a-1)y_4}{mr} +$$
$$+ \frac{4L_2'}{m} - \frac{2\sqrt{2}S_1'}{m} + \frac{2\sqrt{2}S_2'}{m} + 4i\left(\frac{E}{m} + 2\right)y_2 - \frac{2\sqrt{2}y_4'}{m} - \frac{2\sqrt{2}y_5'}{m} = 0,$$

$$\frac{\sqrt{2}S_1(-3a+4imr-3)}{mr} - \frac{\sqrt{2}S_2(3a+4imr+3)}{mr} + \frac{\sqrt{2}y_4(a-8irE+5)}{mr} + \frac{\sqrt{2}y_5(3a-8irE+3)}{mr} +$$
$$+ \frac{4(a+2)y_2}{mr} + \frac{4by_1}{mr} + \frac{4iEL_2}{m} - \frac{2\sqrt{2}S_1'}{m} - \frac{2\sqrt{2}S_2'}{m} + \frac{4y_2'}{m} + \frac{2\sqrt{2}y_4'}{m} + \frac{2\sqrt{2}y_5'}{m} = 0,$$

$$\frac{2\sqrt{2}(a-1)L_2}{mr} + S_1\left(\frac{2-2a}{mr} + 2i\right) + S_2\left(\frac{2(a-1)}{mr} + 2i\right) - \frac{2y_4(a+2ir(2m+E)-1)}{mr} - \frac{2y_5(a+2ir(2m+E)-1)}{mr} -$$
$$- \frac{2\sqrt{2}bL_1}{mr} - \frac{4i\sqrt{2}(2m+E)y_2}{m} = 0.$$

When performing the nonrelativistic approximation, it should be assumed that the order of smallness of variables is governed by relations:

$$L:\ 1,\quad y:\ x,\quad S:\ x,\quad \frac{1}{m}\frac{d}{dr}:\ x,\quad \frac{1}{rm}:\ x,$$
$$rm:\ \frac{1}{x},\quad r\frac{1}{m}\frac{d^2}{dr^2}:\ x,\quad \frac{E}{m}:\ x^2,\quad rE:\ x,\quad r\frac{d}{dr}:\ 1.$$

In the above equations, we preserve the quantities of the leading order. Let us take into account the obtained relations $y_5 = -y_4,\ S_1 = S_2$ into equations

$$\frac{2\sqrt{2}bL_1}{mr}+\frac{2\sqrt{2}(a+5)L_2}{mr}+\frac{4\sqrt{2}L_2'}{m}+4i\sqrt{2}y_2-4iy_4=0,$$

$$-\frac{4iEL_1}{m}+\frac{4y_1'}{m}+\frac{4y_1}{mr}+\frac{2\sqrt{2}bS_1}{mr}+\frac{2\sqrt{2}by_4}{mr}=0,\quad \frac{4L_1}{mr}+\frac{4L_1'}{m}+\frac{4bL_2}{mr}-8iy_1=0,$$

$$-\frac{12iEL_2}{m}+\frac{2\sqrt{2}(a+1)S_1}{mr}-\frac{4\sqrt{2}S_1'}{m}-\frac{4by_1}{mr}-\frac{2\sqrt{2}(a-1)y_4}{mr}-\frac{4ay_2}{mr}+\frac{4y_2'}{m}=0,$$

$$\frac{4(a+2)L_2}{mr}+\frac{4L_2'}{m}-4i\sqrt{2}S_1+8iy_2-8i\sqrt{2}y_4=0,\quad -\frac{2\sqrt{2}bL_1}{mr}+\frac{2\sqrt{2}(a-1)L_2}{mr}+4iS_1-8i\sqrt{2}y_2=0.$$

From the equations without the derivatives of small variables, we eliminate the quantities S_1, y_1, y_2, y_4, and substitute these expressions into two remaining equations, as a result we obtain

$$L_1''+\frac{2}{r}L_1'-\frac{b^2}{r^2}L_1+2mEL_1-\frac{3b}{r^2}L_2=0,$$
$$L_2''+\frac{2}{r}L_2'-\frac{2a(a+3)+b^2+10}{3r^2}L_2+2mEL_2-\frac{b}{r^2}L_1=0.$$

Thus, two coupled 2nd order differential equations for the variables L_1, L_2 are derived; this system describes a spin 3/2 particle in spherically symmetric case.

Let us take into account the presence of the Coulomb field by the formal change $E \Rightarrow E+\alpha/r$ (we group the terms in a special way):

$$[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+2m(E+\frac{\alpha}{r})]L_1=\frac{1}{r^2}[b^2L_1+3bL_2],$$
$$[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+2m(E+\frac{\alpha}{r})]L_2=\frac{1}{r^2}[\frac{2a(a+3)+b^2+10}{3}L_2+bL_1].$$

After performing the transformation, we have the separate equations for the functions $\bar{L}_1, \bar{L}_2$:

$$[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+2m(E+\frac{\alpha}{r})-\frac{(j+2)^2-1/4}{r^2}]\bar{L}_1=0,\quad [\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}+2m(E+\frac{\alpha}{r})-\frac{j^2-1/4}{r^2}]\bar{L}_2=0,$$

they have the same structure, they have the following solutions

$$E_2=-\frac{\alpha^2 m}{2N^2}=-\frac{\alpha^2 m}{2(j+1/2+n)^2},\quad \bar{L}_2=r^{j-1/2}e^{-\sqrt{-2mE}r}F(-n,2j+1,x).$$

$$E_1=-\frac{\alpha^2 m}{2N^2}=-\frac{\alpha^2 m}{2(j+5/2+n)^2},\quad \bar{L}_1=r^{j+3/2}e^{-\sqrt{-2mE}r}F(-n,2j+5,x).$$

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PROBABILITY OF SAFE OPERATION AND FAILURE OF THE TRACTION TRANSFORMER

Traction transformers are part of the converter of the locomotive. Their main function is to reduce the mains voltage to the voltage U_2 required to power traction motors in current receivers. Voltage U_2 is used to adjust the operating voltage of Uv and the motor to the allowable rectified voltage [1–7].

Traction transformers perform the following functions:

- Reduces the mains voltage to the level of the traction motor and other electrical equipment;
- Controls the output voltage according to the given norms;
- Provides energy to the power system of passenger cars [2–5].

Traction transformers in electric locomotives differ from conventional transformers by a very wide range of output voltage control.

Traction oil transformers, designed to reduce the AC voltage to the required level, play an important role in providing reliable and uninterruptible power supply to railway transport. During operation, many faults are observed in traction transformers.

The flawless operation of transformers is one of the most important and most expensive stages of any power system.

It uses many technologies to determine the technical condition of transformers and to predict possible faults in them [3–9].

It includes the following technologies:

- Inspections of transformer oils (chromatographic, laboratory);
- Thermal Inspections;
- Technical inspections, etc.

There are also other forecasting methods, such as visual analysis, inspections, status monitoring, and statistical control methods of processes.