2. Georges, M.P. Real-time holographic interferometry using sillenite photorefractive crystals. Study and optimization of a transportable set-up for quantified phase measurements on large objects / M.P. Georges, Ph.C. Lemaire // Appl. Phys. B. – 1999. – Vol. 68. – P. 1073–1083.

3. Адаптивная интерферометрия, использующая динамические отражательные голограммы в кубических фоторефрактивных кристаллах / А.А. Колегов [и др.] // Квант. электрон. – 2011. – Т. 41, № 9. – С. 847–852.

4. Навныко, В.Н. Влияние поляризации считывающей волны и циркулярного дихроизма на дифракционную эффективность отражательной голограммы в кубическом оптически активном фоторефрактивном поглощающем пьезокристалле / В.Н. Навныко, В.В. Шепелевич, С.М. Шандаров // Оптика и спектроскопия. – 2021. – Т. 129, № 1. – С. 66–74.

5. Определение материальных параметров фоторефрактивных кристаллов на основе метода адаптивной голографической интерферометрии / С.М. Шандаров [и др.] // Оптика и спектроскопия. – 2021. – Т. 129, № 4. – С. 413–417.

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NONRELATIVISTIC APPROXIMATION IN THE PAULI – FIERZ THEORY FOR A SPIN 3/2 PARTICLE IN PRESENCE OF EXTERNAL FIELDS

In the present paper we derive the nonrelativistic equation for spin 3/2 particle in presence of external electromagnetic and gravitational fields.

We start with the relativistic system of equations for wave function with transformation properties of vector-bispinor

$$\gamma^5 \mu_k{}^{can} \gamma_c \left[i(D_a)_n{}^l - \frac{1}{2} M \gamma_a \delta_n{}^l \right] \Psi_l = 0, \qquad (1)$$

$$M = mc/\hbar, \quad D_a = e^{\alpha}_{(a)}(\partial_\alpha + ieA_\alpha) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]a}.$$

With the use of six matrices $\mu_k{}^{can} = (\mu^{[ca]})_k{}^n$, eq. (1) may be presented as follows

$$\gamma^5 (\mu^{[ca]})_k{}^n \gamma_c \left[i(D_a)_n{}^l - M \gamma_a \delta_n{}^l \right] \Psi_l = 0.$$

The above equation may be presented as

$$(Y^0 D_0 + Y^1 D_1 + Y^2 D_2 + Y^3 D_3 + iM)\Psi = 0.$$

First we restrict ourselves to Minkowski space and Cartesian coordinates. The wave function is presented as (the index A is bispinor one, and the (n) is vector one)

$$\Psi_{A(n)} = \begin{vmatrix} f_0 & f_1 & f_2 & f_3 \\ g_0 & g_1 & g_2 & g_3 \\ h_0 & h_1 & h_2 & h_3 \\ d_0 & d_1 & d_2 & d_3 \end{vmatrix}.$$

$$\Psi = \{f_0, g_0, h_0, d_0; f_1, g_1, h_1, d_1; f_2, g_2, h_2, d_2; f_3, g_3, h_3, d_3\}.$$

We can prove that the minimal equation for the matrix $Y^0 = Y_0$ is $Y_0^2(Y_0^2 - 1) = 0$. Therefore, we can define three projective operators

$$P_0 = 1 - Y_0^2, \quad P_1 = P_+ = +\frac{1}{2}Y_0^2(Y+1), \quad P_2 = P_- = -\frac{1}{2}Y_0^2(Y-1).$$

They are found in explicit form, further we get presentation for three projective constituents

$$\Psi_0 = P_0\Psi, \quad \Psi_+ = P_+\Psi, \quad \Psi_- = P_-\Psi .$$

When performing the non-relativistic approximation, we should consider the Ψ_+ as large components, whereas Ψ_- and Ψ_0 as small. Let us consider expressions for large variables. There exist only 4 independent ones:

$$iL_3 = L_6 - L_1, \quad iL_4 = L_5 + L_2 .$$

In accordance with the general method, we are to separate the rest energy: $D_0 \Rightarrow (-iM + D_0)$.

The nonrelativistic approximation is posible in space-times with the following metric

$$dS^2 = (dx^0)^2 + g_{ij}(x)dx^i dx^j , \quad e_{(a)\alpha}(x) = \begin{vmatrix} 1 & 0 \\ 0 & e_{(i)k}(x) \end{vmatrix}; \tag{2}$$

in such models expressions for connections become simpler

$$\Gamma_0 = \frac{1}{2} J^{ik} e^m_{(i)} \, (\nabla_0 e_{(k)m}) , \qquad \Gamma_l = \frac{1}{2} J^{ik} e^m_{(i)} \, (\nabla_l e_{(k)m}) .$$

The contribution of J^{0k} vanishes. Let us derive a generally covariant nonrelativistic equation for the spin 3/2 particle in any space with the structure (2). To this end, we turn to transformation properties of the nonrelativistic wave function under the 3-dimensional rotation group. With this restriction, the transformation rules simplify (2-spinors entering the 4-spinor transform equally)

$$\Psi : \begin{vmatrix} \Phi_{1(0)} & \Phi_{1(1)} & \Phi_{1(2)} & \Phi_{1(3)} \\ \Phi_{2(0)} & \Phi_{2(1)} & \Phi_{2(2)} & \Phi_{2(3)} \\ \varphi_{1(0)} & F_{1(1)} & F_{1(2)} & F_{1(3)} \\ \varphi_{2(0)} & F_{2(1)} & F_{2(2)} & F_{2(3)} \end{vmatrix}, \quad \Psi : \begin{vmatrix} \Phi & \vec{\Phi} \\ \varphi & \vec{F} \end{vmatrix}; \tag{3}$$

so we get $\Phi' = B\Phi, \varphi' = B\varphi, \Phi' = (B \otimes O)\Phi, F' = (B \otimes O)F,$ where the matrices B and O describe 3-rotations for 2-spinors and 3-vectors. In (3), let us preserve only the large components Ψ_+ it suffices to follow only two first rows. For generators:

$$J_j = (i/2)\sigma_j \otimes I + I \otimes V_j, \quad j = 1,2,3,$$

we readily find 6-dimensional representations

$$\Phi = \begin{vmatrix} \Phi_{11} \\ \Phi_{12} \\ \Phi_{13} \\ \Phi_{21} \\ \Phi_{22} \\ \Phi_{23} \end{vmatrix} = \begin{vmatrix} L_1 \\ L_3 \\ L_5 \\ L_2 \\ L_4 \\ L_6 \end{vmatrix}, \quad J_1 = \begin{vmatrix} 0 & 0 & 0 & i/2 & 0 & 0 \\ 0 & 0 & -1 & 0 & i/2 & 0 \\ 0 & 1 & 0 & 0 & 0 & i/2 \\ i/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & i/2 & 0 & 0 & 0 & -1 \\ 0 & 0 & i/2 & 0 & 1 & 0 \end{vmatrix},$$

$$J_2 = \begin{vmatrix} 0 & 0 & 1 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1/2 \\ -1/2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/2 & -1 & 0 & 0 \end{vmatrix}, \quad J_3 = \begin{vmatrix} i/2 & -1 & 0 & 0 & 0 & 0 \\ 1 & i/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & i/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i/2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -i/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i/2 \end{vmatrix}.$$

We find commutators

$$J_1J_2 - J_2J_1 = J_3 + K_3,\ J_2J_3 - J_3J_2 = J_1 + K_1,\ J_3J_1 - J_1J_3 = J_2 + K_2,$$

also we find the identities

$$K_3^2 = -I, K_2^2 = -I, K_1^2 = -I,$$

and

$$K_2K_3 + K_3K_2 = 0,\ K_3K_1 + K_1K_3 = 0,\ K_1K_2 + K_2K_1 = 0,\ K_2K_3 = K_1,\ K_3K_1 = K_2,\ K_1K_2 = K_3.$$

The commutation relations

$$K_2K_3 - K_3K_2 = 2K_1, \quad K_3K_1 - K_1K_3 = 2K_2, \quad K_1K_2 - K_2K_1 = 2K_3,$$

after transition to $S_i = K_i/2$ take the form of the algebra SO(3):

$$S_2S_3 - S_3S_2 = S_1, \quad S_3S_1 - S_1S_3 = S_2, \quad S_1S_2 - S_2S_1 = S_3.$$

We readily obtain the identity

$$(K_1D_1 + K_2D_2 + K_3D_3)^2 = -(D_1^2 + D_2^2 + D_3^2) +$$
$$+K_2K_3(D_2D_3 - D_3D_2) + K_3K_1(D_3D_1 - D_1D_3) + K_1K_2(D_1D_2 - D_2D_1),$$

whence it follows

$$\frac{1}{2M}(K_1D_1 + K_2D_2 + K_3D_3)^2 = -\frac{1}{2M}(D_1^2 + D_2^2 + D_3^2) + \frac{ie}{M}(F_{23}S_1 + F_{31}S_2 + F_{12}S_3), \qquad (4)$$

which has the structure of the non-relativistic Hamiltonian in 6-demensional form. We can prove that (4) indeed reduces to the known nonrelativistic equation for the spin 3/2 particle.

To this end, let us start with the explicit form of equation (4) (let $ie/2M=\mu$, and take into account the constraints $L_3=(iL_1-iL_6), L_4=(-iL_2-iL_5)$). We divide 6 equations into two groups. In each group, only two equations are independent:

$$iD_0(L_1+L_6)=-\frac{1}{2M}D^2(L_1+L_6)+\mu F_{23}(-iL_2-iL_5)+\mu F_{31}(-L_2+L_5)+\mu F_{12}(-iL_1+iL_6),$$
$$iD_0(L_2-L_5)=-\frac{1}{2M}D^2(L_2-L_5)+\mu F_{23}(-iL_1+iL_6)+\mu F_{31}(L_1+L_6)+\mu F_{12}(iL_2+iL_5),$$
$$iD_0(2L_1-L_6)=-\frac{1}{2M}D^2(2L_1-L_6)+\mu F_{23}(-2iL_2+iL_5)+\mu F_{31}(-2L_2-L_5)+\mu F_{12}(-2iL_1-iL_6),$$
$$iD_0(2L_2+L_5)=-\frac{1}{2M}D^2(2L_2+L_5)+\mu F_{23}(-2iL_1-iL_6)+\mu F_{31}(2L_1-L_6)+\mu F_{12}(2iL_2-iL_5).$$

Let us introduce the new 4 components

$$L_1=\frac{1}{3}\Psi_1+\frac{1}{3}\Psi_3,\quad L_2=\frac{1}{3}\Psi_2+\frac{1}{3}\Psi_4,\quad L_5=\frac{1}{3}\Psi_4-\frac{2}{3}\Psi_2,\quad L_6=\frac{2}{3}\Psi_1-\frac{1}{3}\Psi_3.$$

Then the above system takes a new form (we write down it in matrix representation)

$$\Psi=\begin{vmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\\ \Psi_4\end{vmatrix},\quad iD_0\Psi=-\frac{1}{2M}\Delta\Psi+\frac{ie}{3M}(F_{23}S_1+F_{31}S_2+F_{12}S_3)\Psi, \qquad (5)$$

where

$$S_1=\frac{i}{2}\begin{vmatrix}0 & 1 & 0 & -2\\ 1 & 0 & -2 & 0\\ 0 & -2 & 0 & 1\\ -2 & 0 & 1 & 0\end{vmatrix}, S_2=\frac{1}{2}\begin{vmatrix}0 & -3 & 0 & 0\\ 3 & 0 & 0 & 0\\ 0 & -2 & 0 & 1\\ 2 & 0 & -1 & 0\end{vmatrix}, S_3=\frac{i}{2}\begin{vmatrix}1 & 0 & -2 & 0\\ 0 & -1 & 0 & 2\\ 0 & 0 & -3 & 0\\ 0 & 0 & 0 & 3\end{vmatrix}.$$

The last matrices obey the commutation rule $S_1S_2-S_2S_1=S_3$, and so on; therefore, they may be considered as the components of the spin operator.

Now we can generalize the non-relativistic equation (5) to the generally covariant form. The structure of that equation should be as follows (we start with the 6-dimensional form)

$$iD_0\Psi_6=\frac{1}{2M}[K^j(x)(\frac{\partial}{\partial x^j}+\Gamma_j(x)+ieA_j(x))]^2\Psi_6,$$
$$K^j(x)=K_i e^j_{(i)}(x),\quad \Gamma_j(x)=\frac{1}{2}J^{kl}e^n_{(k)}(x)\nabla_j e_{(l)n}(x),$$

where the generalized derivatives are determined by the formulas

$$D_0(x) = \partial_0 + ieA_0(x) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]0}(x),$$

$$D_k(x) = e^j_{(k)}(x)(\partial_j + ieA_j(x)) + \frac{1}{2}(\sigma^{ps} \otimes I + I \otimes j^{ps})\gamma_{[ps]k}(x), k = 1,2,3.$$

The definition of the 6 large components remains the same, two linear constraints preserve their form as well. All algebraic transformations proving existence of only 4 independent equations also are the same. The difference consists only in the new and more complicated expressions for generalized derivatives. Correspondingly, we obtain the generalized equation

$$iD_0(x)\Psi = -\frac{1}{2M}(D_1^2(x) + D_2^2(x) + D_3^2(x))\Psi + \frac{1}{2M}(D_{[23]}S_1 + D_{[31]}S_2 + D_{[12]}S_3)\Psi_4,$$

where the commutators $D_{[kl]} = D_k(x)D_l(x) - D_l(x)D_k(x)$ are used.

REFERENCES

1. Pauli, W. Über relativistische Feldleichungen von Teilchen mit beliebigem Spin im elektromagnetischen Feld / W. Pauli, M. Fierz // Helvetica Physica Acta. – 1939. – Bd. 12. – S. 297–300.
2. Fierz, M. On relativistic wave equations for particles of arbitrary spin in an electromagnetic field / M. Fierz, W. Pauli // Proc. Roy. Soc. London. A. – 1939. – Vol. 173. – P. 211–232.
3. Rarita, W. On a theory of particles with half-integral spin / W. Rarita, J. Schwinger / Phys. Rev. – 1941. – Vol. 60, № 1. – P. 61–64.
4. Elementary particles with internal structure in external fields. I. General theory, II. Physical problems / V.V. Kisel [et al.]. – New York : Nova Science Publishers Inc., 2018. – P. 402, 404.

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SPIN 3/2 PARTICLE IN THE COULOMB FIELD, TETRAD FORMALISM, NONRELATIVISTIC APPROXIMATION

In the present paper, we will study the nonrelativistic approximation for a spin 3/2 particle [1–3] in the external Coulomb field. In [4] (see also [5; 6]), the nonrelativistic equation was derived from the relativistic Pauli – Fierz theory; in this case, the Cartesian coordinates were used and the presence of an arbitrary electromagnetic field was taken into account. In the present paper, the system of radial equations describing the nonrelativistic spin 3/2 particle in the Coulomb field will be obtained. Exact solutions are found in terms of the confluent hypergeometric functions, and the corresponding energy spectra are obtained.

In [4; 5], the system of radial equations for spin 3/2 particle in absence of external fields was derived. The substitution for the wave function has the form [6]

$$\Psi = \Psi_{A(l)} = e^{-imt} \begin{vmatrix} f_0 D_{-1/2} & f_1 D_{-3/2} & f_2 D_{-1/2} & f_3 D_{+1/2} \\ g_0 D_{+1/2} & g_1 D_{-1/2} & g_2 D_{+1/2} & g_3 D_{+3/2} \\ h_0 D_{-1/2} & h_1 D_{-3/2} & h_2 D_{-1/2} & h_3 D_{+1/2} \\ d_0 D_{+1/2} & d_1 D_{-1/2} & d_2 D_{+1/2} & d_3 D_{+3/2} \end{vmatrix}, \qquad (1)$$